

Chapter 5

X-RAY STANDING WAVE AT THE TOTAL REFLECTION CONDITION

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Fresnel theory is used to derive the complex electric fields above and below an X-ray– reflecting interface that separates two materials with differing refraction indices. The interference between the incident and reflected waves produces an X-ray standing wave (XSW) above the reflecting interface. The XSW intensity modulation is strongly enhanced by the total external reflection condition, which occurs at incident angles less than the critical angle. At these small milliradian incident angles, the XSW period ($\lambda/2\theta$) becomes very large, which makes the TR-XSW an ideal probe for studying low-density structures that extend 1 to 1000 nm above the reflecting interface.

5.1. Introduction

The original (and most widely used) method for generating an X-ray standing wave (XSW) ~~is using~~ dynamical diffraction from a perfect single crystal in a Bragg reflection geometry.^{1–3} However, as with any standing wave phenomena, the minimum requirement is the superposition of two coherently coupled plane waves. Therefore, one can imagine several alternative geometries for generating an XSW. This chapter discusses the case of generating an XSW by total external reflection from an X-ray mirror surface.⁴

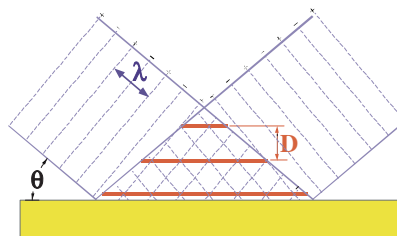


Fig. 5.1. Illustration of XSW generated by interference between incident and specular reflected plane waves.

1 Referring to Fig. 5.1, the primary distinguishing feature for the total-
 2 reflection (TR) case is that the length of the XSW period above the
 3 mirror surface,

$$D = \frac{\lambda}{2 \sin \theta} = \frac{2\pi}{Q}, \quad (5.1)$$

4 is much longer, since TR occurs at very small incident angles, θ . Also, the
 5 length of the XSW period, D , will continuously decrease as θ increases
 6 through the range of TR. This long-period XSW is ideally suited
 7 for measuring surfaces, interfaces, and supported nanostructures with
 8 structural features that range from 50 to 2000 Å. Examples include studies
 9 of Langmuir–Blodgett (LB) multilayers,^{4–7} layer-by-layer self-assembly of
 10 metal–organic films,^{8,9} the diffuse double-layer formation at the electrified
 11 water/solid interface,^{10,11} biofilm ion adsorption,¹² and metal nanoparticle
 12 dispersion in polymer films.^{13,14}

13 5.2. X-Ray Transmission and Reflection 14 at a Single Interface

15 Based on Maxwell’s equations, an electromagnetic traveling plane-wave
 16 impinging on a boundary separating two different refractive media, splits
 17 into a reflected and transmitted (or refracted) plane-wave.¹⁵ At X-ray
 18 frequencies, the index of refraction,

$$n_j = 1 - \delta_j - i\beta_j \quad (5.2)$$

19 is less than unity and therefore (as illustrated in Fig. 5.2) the angle of
 20 refraction, θ_2 , is less than the incident angle, θ_1 .¹⁶ Parameters δ_j and β_j ,
 21 which account, respectively, for refraction and absorption effects by the

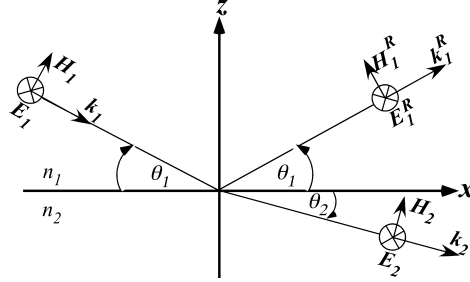


Fig. 5.2. The σ -polarization case for the reflection and refraction of X-rays at a boundary separating two media with indices of refraction $n_1 > n_2$.

1 j th medium, can be expressed as:

$$\delta = -\frac{1}{2}\chi'_0 = \frac{r_e\lambda^2}{2\pi}N'_e, \quad (5.3)$$

$$\beta = -\frac{1}{2}\chi''_0 = \frac{\lambda\mu_0}{4\pi}, \quad (5.4)$$

2 where N'_e is the real part of the effective electron density.

3 The E-fields associated with the incident, reflected, and transmitted
4 plane-waves are expressed respectively as:

$$\bar{\epsilon}_1(\mathbf{r}, t) = \mathbf{E}_1 \exp(-i(\mathbf{k}_1 \cdot \mathbf{r} - \omega t)) \quad (5.5a)$$

$$\bar{\epsilon}_1^R(\mathbf{r}, t) = \mathbf{E}_1^R \exp(-i(\mathbf{k}_1^R \cdot \mathbf{r} - \omega t)) \quad (5.5b)$$

$$\bar{\epsilon}_2(\mathbf{r}, t) = \mathbf{E}_2 \exp(-i(\mathbf{k}_2 \cdot \mathbf{r} - \omega t)) \quad (5.5c)$$

5 At $z = 0$, the space and time variations of all three fields must be equivalent.
6 This produces the “law of co-planarity,” which requires the transmitted and
7 reflected wave-vectors, \mathbf{k}_2 and \mathbf{k}_1^R , to be confined to the same plane as the
8 incident wave-vector, \mathbf{k}_1 (the xz -plane in Fig. 5.2). The continuity of the
9 tangential components of the three wave-vectors at the boundary dictates
10 the kinematical properties corresponding to the “law of reflection” $\theta_1^R = \theta_1$
11 and the “law of refraction” (Snell’s Law) $n_2 \cos \theta_2 = n_1 \cos \theta_1$. Using these
12 relationships the spatial components in Eq. (5.5) can be expressed as:

$$\mathbf{k}_1 \cdot \mathbf{r} = k_1(x \cos \theta_1 - z \sin \theta_1) \quad (5.6a)$$

$$\mathbf{k}_1^R \cdot \mathbf{r} = k_1(x \cos \theta_1 + z \sin \theta_1) \quad (5.6b)$$

$$\mathbf{k}_2 \cdot \mathbf{r} = \frac{n_2}{n_1}k_1 \left[x \frac{n_1}{n_2} \cos \theta_1 - z \sqrt{1 - \left(\frac{n_1}{n_2} \cos \theta_1 \right)^2} \right] \quad (5.6c)$$

1 Total reflection occurs when the transmitted plane-wave $\bar{\epsilon}_2(\mathbf{r}, t)$ pro-
 2 pagates strictly in the x -direction and is attenuated in the inward negative
 3 z -direction. From Eqs. (5.5c) and (5.6c), TR occurs when $\theta_1 < \theta_C$. For
 4 $n_1 = 1$ (e.g., vacuum or air) and $n_2 = 1 - \delta - i\beta$, the critical angle¹⁶ is

$$\theta_C = \sqrt{2\delta}. \quad (5.7)$$

5 The scattering vector at the critical angle is

$$Q_c = 4\pi \sin \theta_c / \lambda \cong 4\pi \theta_c / \lambda = 4\sqrt{\pi r_e N'_e}. \quad (5.8)$$

6 If dispersion corrections are ignored, $N'_e = N_e$ and Q_c becomes a
 7 wavelength-independent property, in which case, $Q_c = 0.0315 \text{ \AA}^{-1}$ for Si
 8 and $Q_c = 0.0812 \text{ \AA}^{-1}$ for Au.

9 The continuity of the tangential components of the E-fields and
 10 magnetic-fields at the $z = 0$ boundary dictates the dynamical properties
 11 of the fields, corresponding to the Fresnel equations, which for the
 12 σ -polarization case and for small angles θ_1 can be expressed as

$$F_{1,2}^R = \frac{E_1^R}{E_1} = \left| \frac{E_1^R}{E_1} \right| e^{iv} = \frac{q - \sqrt{q^2 - 1 - ib}}{q + \sqrt{q^2 - 1 - ib}} \quad (5.9)$$

$$F_{1,2}^T = \frac{E_2}{E_1} = \frac{2q}{q + \sqrt{q^2 - 1 - ib}}, \quad (5.10)$$

13 where the normalized angle $q = \theta_1 / \theta_C = Q / Q_c$ and $b = \beta / \delta$ for the case of
 14 $n_1 = 1$ and $n_2 = 1 - \delta - i\beta$. At $\lambda = 0.71 \text{ \AA}$, $b = 0.005$ for Si and $b = 0.1$
 15 for Au.

16 Figure 5.3 shows the q dependence of the reflectivity, $R = |F_{1,2}^R|^2$, the
 17 normalized E-field intensity at the surface, $I_{z=0} = |F_{1,2}^T|^2$, the phase of
 18 the reflected plane-wave relative to the phase of the incident at $z = 0$,
 19 $v = \text{Arg}(F_{1,2}^R)$, and the penetration depth

$$\Lambda = 1/\mu_e = \left[Q_c \text{Re} \left[\sqrt{1 - q^2 + ib} \right] \right]^{-1}. \quad (5.11)$$

20 As can be seen, TR occurs for $q < 1$, where the reflectivity approaches
 21 unity, the phase shifts by π radians, and E-field intensity below the surface
 22 forms an evanescent wave¹⁷ with a penetration depth approaching Q_c^{-1} ,
 23 which is 32 \AA for Si and 12 \AA for Au. For q increasing above unity, the
 24 reflectivity quickly reduces (approaching zero) and the transmitted wave
 25 propagates into the medium with a penetration depth quickly approaching
 26 the normal absorption process, where $\Lambda = \sin \theta / \mu_0$.

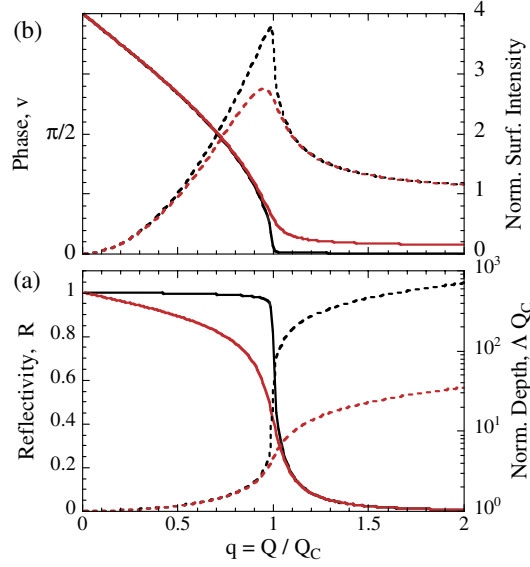


Fig. 5.3. (Color online) The Fresnel theory calculated, normalized-angular dependence of the (a) reflectivity (solid lines) and normalized penetration depth (dashed lines) and (b) phase (solid lines) and normalized surface E-field intensity (dashed lines). The black colored curves are for the weak absorption-case of $b = 0.005$ and the red curves are for $b = 0.1$.

1 An equivalent expression for the complex reflectivity amplitude of
 2 Eq. (5.9) can be derived from dynamical diffraction theory by solving the
 3 symmetric zeroth-order Bragg diffraction condition, i.e., set the structure
 4 factor $F_H = F_0$ in the expression for angle parameter η . This equivalence
 5 is simply due to the fact that TR is the zeroth-order dynamical Bragg
 6 diffraction condition, where the d -spacing is infinite.

7 5.3. The E-Field Intensity

8 The total E-field in the vacuum (or air) above the mirror surface, where the
 9 incident and reflected plane waves are coherently coupled by $\mathbf{Q} = \mathbf{k}_1^R - \mathbf{k}_1$,
 10 is expressed as $\bar{\varepsilon}_T = \bar{\varepsilon}_1 + \bar{\varepsilon}_1^R$, and below the mirror surface, $\bar{\varepsilon}_T = \bar{\varepsilon}_2$. The
 11 E-field intensity, $I = |\bar{\varepsilon}_T|^2$, can then be expressed as

$$I(\theta, z) = I_0 \begin{cases} 1 + R + 2\sqrt{R} \cos(v - Qz), & \text{for } z \geq 0 \\ |F_{1,2}^T|^2 \exp(-\mu_e |z|), & \text{for } z \leq 0 \end{cases}, \quad (5.12)$$

1 where $I_0 = |E_1|^2$ is the intensity of the incident plane wave and μ_e is the
 2 effective linear absorption coefficient defined in Eq. (5.11). As can be seen
 3 in Fig. 5.4, the E-field intensity under the TR condition exhibits a standing
 4 wave above the mirror surface with a period $D = 2\pi/Q$ and an evanescent
 5 wave below the surface. The height coordinate in Fig. 5.4 is normalized
 6 to the critical period $D_c = 2\pi/Q_c$, which is 199 Å for Si and 77 Å for Au
 7 (if $\Delta f'' = \Delta f' = 0$).

8 As can be seen from Figs. 5.3 and 5.4, at $q = 0$, there is a node in the
 9 E-field intensity at the mirror surface and the first antinode is at infinity.
 10 As q increases, that first antinode moves inward and reaches the mirror
 11 surface at $q = 1$. This inward movement of the first antinode, which is
 12 analogous to the Bragg diffraction case, is due to the π phase-shift depicted
 13 in Fig. 5.3(b). The other XSW antinodes follow the first antinode with a
 14 decreasing period of $D = 2\pi/Q$. For q increasing above unity, the XSW
 15 phase is fixed, the period D continues to contract, and the XSW amplitude
 16 drops off dramatically.

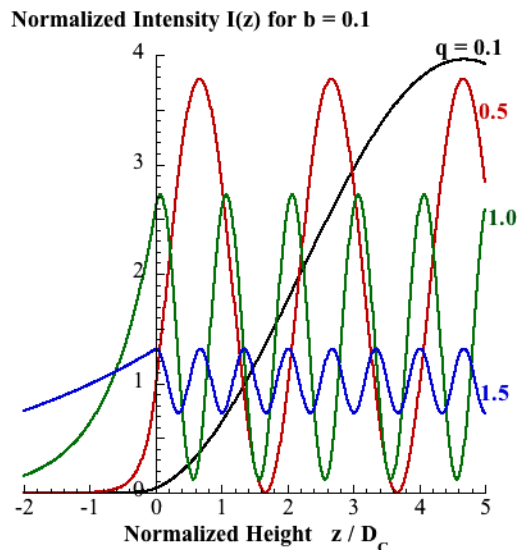


Fig. 5.4. The normalized-height dependence of the normalized E-field intensity for different normalized incident angle q . An XSW exists above the mirror surface and an evanescent wave exists below the surface for $q < 1$. The calculations is for the case of $b = 0.1$ in Eq. (5.12).

5.4. X-Ray Fluorescence Yield from an Atomic Layer within a Thin Film

The q dependence for the normalized E-field intensity at $z = 0$ is shown in Fig. 5.3(b). Figure 5.5 shows the Eq. (5.12) calculation for the two additional heights above the surface. These three curves illustrate the basis for the TR-XSW technique as a positional probe, since (in the dipole approximation for the photo-effect) the XRF yield, $Y(q)$, from an atomic-layer at a discrete height z will follow such a curve. Note that in the TR range, $0 < q < 1$, the number of modulations in the E-field intensity is equivalent to $z/D_c + (1/2)$. The extra $1/2$ modulation is due to the π phase shift shown in Fig. 5.3(b). Therefore, for an XRF-marker atom layer within a low-density film on a high-density mirror, the atomic layer height can be quickly approximated by counting the number of modulations in the XRF yield that occur between the film critical angle and the mirror critical angle. Referring to Fig. 5.6, this effect can be seen in the experimental results and analysis for the case of a Zn atomic layer trapped at the topmost bilayer of a 1000-Å-thick LB multilayer that was deposited on a gold mirror. There are $11\frac{1}{2}$ Zn $K\alpha$ XRF modulations as the incident angle is advanced over this range, indicating that the Zn layer is at a height of 11 critical periods (or 900 Å) above the gold surface. From the simultaneously collected reflectivity shown in Fig. 5.6, the critical angles for the LB film and Au mirror are at 2.15 and 7.52 mrad, respectively. A more rigorously determined Zn atomic distribution profile, $\rho(z)$, is determined by a fit of

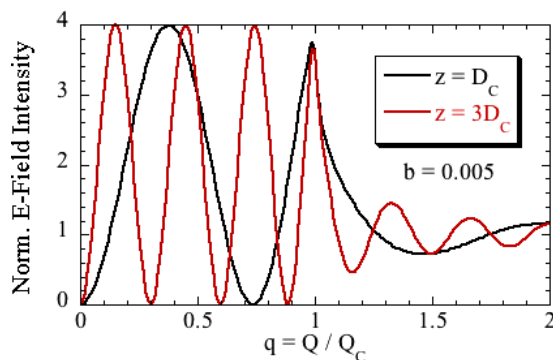


Fig. 5.5. The normalized-angle dependence of the normalized E-field intensity for two different heights above the mirror surface. The calculations are for the case of $b = 0.005$ in Eq. (5.12). The critical period $D_C = 199$ Å for Si and 77 Å for Au.

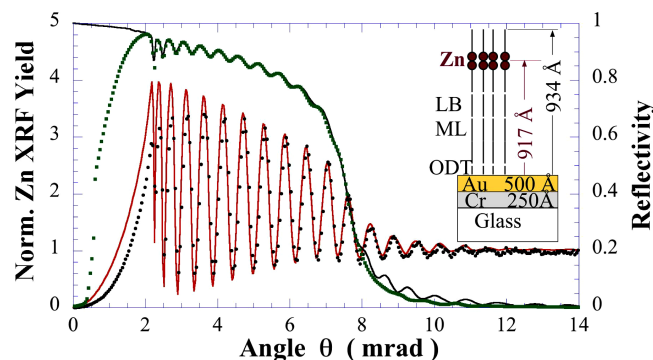


Fig. 5.6. The experimental and theoretical reflectivity and Zn $K\alpha$ XRF yield versus incident angle at $\lambda = 1.215 \text{ \AA}$ from a LB multilayer-coated gold mirror depicted in the inset. From the reflectivity fit, the film thickness is measured to be $t_F = 934 \text{ \AA}$ and the interface roughness $\sigma_1 = 3 \text{ \AA}$. From the XSW Zn yield fit with a modeled Zn Gaussian distribution, $\langle z \rangle = 917 \text{ \AA}$ and $\sigma_{Zn} = 15 \text{ \AA}$ ($FWHM = 35.3 \text{ \AA}$). The data deviation from theory for $\theta < 2 \text{ mrad}$ is due to x-ray footprint geometrical effects. See Ref. 5, for details from a similar measurement on a similar sample.

1 the modeled XRF yield $Y(\theta) = \int \rho(z)I(\theta, z)dz$ to the data in Fig. 5.6,
 2 where the E-field intensity $I(\theta, z)$ within the refracting (and absorbing)
 3 film was calculated by an extension of Parratt's recursion formulation¹⁸
 4 described in the section entitled "XSW in Multilayers" (Chapter 7). This
 5 same model described in the inset was also used to generate a fit to the
 6 reflectivity data (Fig. 5.6), which is independently sensitive to the density
 7 and thickness of the film and the widths of the interfaces. The very sharp
 8 drop in the reflectivity at the film critical angle (2.15 mrad) is due to the
 9 excitation of the first mode of a resonant cavity that was observed to
 10 produce a 20-fold enhancement in the E-field intensity at the center of the
 11 film.¹⁹

12 5.5. Fourier Inversion for a Direct Determination of $\rho(z)$

13 Similar to the Bragg diffraction XSW case (see the section entitled "XSW
 14 Imaging", Chap. 14), the TR-XSW XRF yield is also directly linked to
 15 the Fourier transform of the atomic distribution. In this case, however, the
 16 Fourier transform is measured at low- Q , over a continuous range in Q and
 17 only in the Q_z direction. This section describes how the Fourier transform
 18 can be extracted from the TR-XSW data to produce a model-independent
 19 measure of the atomic distribution profile, $\rho(z)$.

1 To account for the refraction and absorption effects that will influence
 2 the observed reflectivity and XRF yields from a film-coated mirror, the
 3 earlier described two-layer model (Fig. 5.2) needs to be replaced by a three-
 4 layer model (or double interface model) formed by vacuum (the $j = 1$ layer),
 5 a thin low-density film ($j = 2$), and a higher-density mirror ($j = 3$). The
 6 $j = 2$ film/mirror interface is at $z = 0$ and the $j = 1$ vacuum/film interface
 7 is at $z = t_F$. For the present case study, $\delta_1 = \beta_1 = 0$, $\delta_2 \ll \delta_3$, and $\beta_2 \ll$
 8 $\beta_3 \ll \delta_3$. For the Fig. 5.6 example of 1.215 Å X-rays reflecting from a gold
 9 mirror coated with a LB multilayer, $\delta_2 = 2.31 \times 10^{-6}$, $\delta_3 = 2.83 \times 10^{-5}$,
 10 $\beta_2 = 1.90 \times 10^{-9}$, $\beta_3 = 1.9610^{-6}$, $\theta_{c,2} = 2.15$ mrad, $\theta_{c,3} = 7.52$ mrad. At
 11 $\theta_1 = \theta_{c,3}$, the XSW period is 80.8 Å in the vacuum and 84.2 Å inside the
 12 LB film.

13 TR exists at the interface above the j th layer when $\theta_1 < \theta_{c,j} = (2\delta_j)^{1/2}$.
 14 When $\theta_1 > \theta_{c,2}$, a refracted (or transmitted) traveling wave penetrates
 15 through the film and is reflected from the mirror surface. The total E-field
 16 intensity within the film is then described as

$$I_2^T(q_2, Z) = I_2(q_2, Z) + I_2^R(q_2, Z) + 2\sqrt{I_2(q_2, Z) I_2^R(q_2, Z)} \\ \times \cos(v_2(q_2) - 2\pi q_2 Z), \quad (5.13)$$

17 where I_2 and I_2^R are the respective intensities of the incident (transmitted)
 18 and reflected plane waves that form an interference field within the film.
 19 The refraction-corrected normalized angle (or normalized scattering vector)
 20 within the film is defined as

$$q_2 = Q_2/Q_{c,2} = (\theta_1^2 - \theta_{c,2}^2)^{1/2} (\theta_{c,3}^2 - \theta_{c,2}^2)^{-1/2}. \quad (5.14)$$

21 Here Z is the normalized height above the mirror surface in units of the
 22 refraction corrected critical period. Namely, $Z = z/D_{c,2}$, where $D_{c,2} =$
 23 $\lambda_1/(2\theta_{c,3})/(1 - (1/2)\delta_2/\delta_3)$. The use of generalized coordinates q_2 and Z
 24 makes the description independent of wavelength and index of refraction.
 25 The phase of the reflected plane wave relative to the incident at $z = Z = 0$
 26 is expressed as v_2 . Based on the dipole approximation for the photoelectric
 27 effect, the fluorescence yield from a normalized atomic distribution $\rho(Z)$
 28 within the film is

$$Y(q_2) = \int_0^{t'_F} I_2^T(q_2, Z) \rho(Z) dZ, \quad (5.15)$$

29 where $t'_F = t_F/D_{c,2}$ is the normalized film thickness.

1 **Weakly Absorbing Film Approximation:** If the attenuation depth of
 2 I_2 and I_2^R within the film is large in comparison to the spread $\langle Z^2 \rangle^{1/2}$ of
 3 $\rho(Z)$, then Eq. (5.15) can be simplified, so that the yield for a normalized
 4 incident intensity and normalized distribution is expressed as

$$Y(q_2) = I_2(q_2, \langle Z \rangle) + I_2^R(q_2, \langle Z \rangle) + 2\sqrt{I_2(q_2, \langle Z \rangle)I_2^R(q_2, \langle Z \rangle)}y(q_2), \quad (5.16)$$

5 where the modulation in the yield due to the interference fringe field is

$$y(q_2) = \int_0^{t'_F} \rho(Z) \cos(v_2(q_2) - 2\pi q_2 Z) dZ. \quad (5.17)$$

6 Since $I_2(q_2, \langle Z \rangle)$ and $I_2^R(q_2, \langle Z \rangle)$ can be calculated from Parratt's recursion
 7 formulation, this reduced yield, $y(q_2)$, can be extracted from the measured
 8 yield $Y(q)$. Figure 5.7 shows this for the yield data shown in Fig. 5.6.
 9 The inverse Fourier transform of this reduced yield can be directly used
 10 to generate the fluorescence selected atom distribution $\rho(z)$ to within a
 11 resolving limit defined by the range of Q over which the visibility of the
 12 interference fringes is significant.

13 **Linear Phase Approximation:** If the phase v_2 were zero in Eq. (5.17),
 14 $y(q_2)$ would simply be the real part of the Fourier transform of $\rho(Z)$. Based

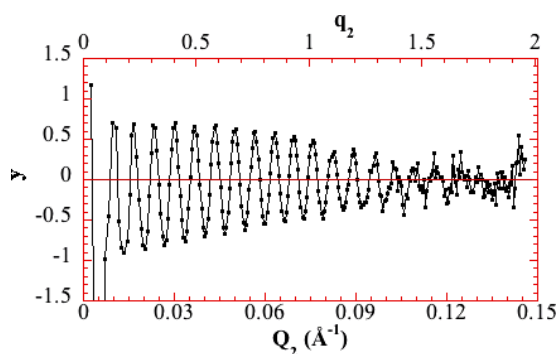


Fig. 5.7. The reduced yield that was extracted from the Zn γ yield data in Fig. 6. See Eqs. (5.16) and (5.17). The line connecting data points is drawn to guide the eye. The oscillation period and envelop width are inversely related to the mean height and intrinsic width, respectively, of the Zn distribution profile.

1 on Fresnel theory for the case of no absorption ($\beta = 0$) the phase is

$$v_2(q_2) = \begin{cases} \cos^{-1}(2q_2^2 - 1), & \text{for } 0 \leq q_2 < 1 \\ 0, & \text{for } q_2 \geq 1 \end{cases}. \quad (5.18)$$

2 As can be seen in Fig. 5.3(b), v can be reasonably approximated by a linear
3 function in the TR region as: $v(q) \sim (1 - q)\pi$, for $0 \leq q < 1$. Introducing
4 this approximation into Eq. (5.17), simplifies the expression for the reduced
5 yield to

$$y(q_2) = \begin{cases} -\int_0^1 \rho(Z) \cos[2\pi q_2(Z + 1/2)] dZ, & \text{for } 0 \leq q_2 < 1 \\ \int_0^1 \rho(Z) \cos[2\pi q_2 Z] dZ, & \text{for } q_2 \geq 1 \end{cases}. \quad (5.19)$$

6 The atomic density profile can then be directly generated from the TR-
7 XSW data as

$$\rho(Z) = \sum_{q_2 > 0} s(q_2) y(q_2) \cos[2\pi q_2(Z + \delta(q_2))] \Delta q_2, \quad (5.20)$$

8 where $s = -1$ and $\delta = 1/2$ for $0 < q_2 < 1$ and $s = 1$ and $\delta = 0$ for
9 $q_2 > 1$. In Fig. 5.8 this is illustrated for the data in Figs. 5.6 and 5.7.
10 The resolution for this model-independent Fourier inversion of this data is
11 $\pi/Q_{2,\max} = 25 \text{ \AA}$. The precision for the height and width of a Gaussian
12 model fit to this type of data is typically ± 2 to $\pm 5 \text{ \AA}$ (see Refs. 4 and 5).

13 5.6. The Effect of Coherence on X-Ray Interference Fringe 14 Visibility

15 If the spatial and temporal coherence properties of the incident photon
16 beam are well known, the TR-XSW observation described above can be
17 used to determine the spatial distribution of the fluorescent atom species
18 within the film. Conversely, if the spatial distribution of the fluorescent
19 atomic species is known, the observation of the X-ray interference fringes
20 can be used to characterize the longitudinal and transverse coherence
21 lengths of the incident photon beam. This is demonstrated in Fig. 5.9,
22 where three separate TR-XSW measurements are taken with three different
23 longitudinal coherence lengths ($L_L = \lambda^2/\Delta\lambda$) of the same LB multilayer
24 structure described in the inset of Fig. 5.6. The fringe visibility, as observed
25 by the Zn $K\alpha$ fluorescence, is not affected by a reduction in L_L until

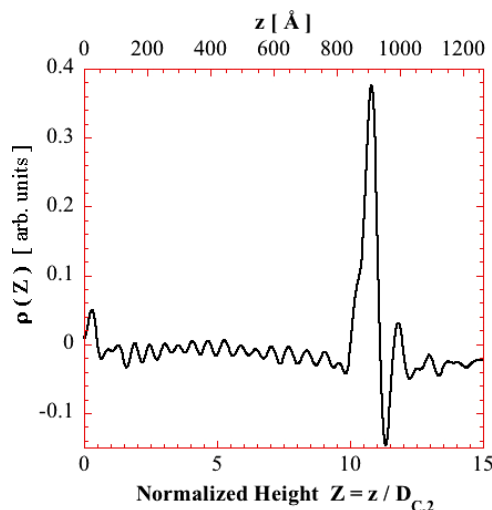


Fig. 5.8. The Zn distribution profile directly generated by the Fourier inversion of the reduced Zn $K\alpha$ XRF yield data from Fig. 7. The summation in Eq. (5.20) was for data in the range $0.09 < q_2 < 1.91$. The peak at $Z = 10.8$ or $z = 909 \text{ \AA}$ has a $FWHM = 43 \text{ \AA}$. This corresponds to the convolution of the intrinsic width (35 \AA) with the resolution width (25 \AA). The truncation-error oscillations have a period of 50 \AA corresponding to an effective Q_2 range of 0.13 \AA^{-1} .

1 the optical path-length difference (in units of λ) between the two beams
 2 at the Zn height (expressed as $Q_2\langle z \rangle/2\pi$) approaches the value of the
 3 monochromaticity, $\lambda/\Delta\lambda$. Referring to Fig. 5.10, the optical path-length
 4 difference is $n(BC - AC) = n(2z \sin\theta)$. In Fig. 5.9, the top curve
 5 (identical to Fig. 5) corresponds to a nearly ideal plane wave condition
 6 produced by using a $Si(111)$ monochromator. The lower two curves
 7 correspond to much wider band-pass incident beams that were prepared
 8 by Bragg diffraction from two different multilayer monochromators (Si/Mo
 9 and C/Rh).

10 A reduction in the interference fringe visibility due to a limited
 11 transverse coherence should not occur if the transverse coherence length
 12 $L_T \gg z$. Therefore, in consideration of typical longitudinal and transverse
 13 coherence lengths at third generation SR undulator beamlines, the
 14 TR-XSW method that uses 1-\AA wavelength X-rays should be extendable
 15 as a probe to a length-scale of $1 \mu\text{m}$ above the mirror surface.

16 It is worth noting that the reduced yield $y(q)$, as defined in Eqs. (5.16)
 17 and (5.17) and shown measured and Fourier inverted in Figs. 5.6 and 5.7,
 18 is a measurement of the real part of the degree of coherence $\text{Re}[\gamma_{12}]$.¹⁵

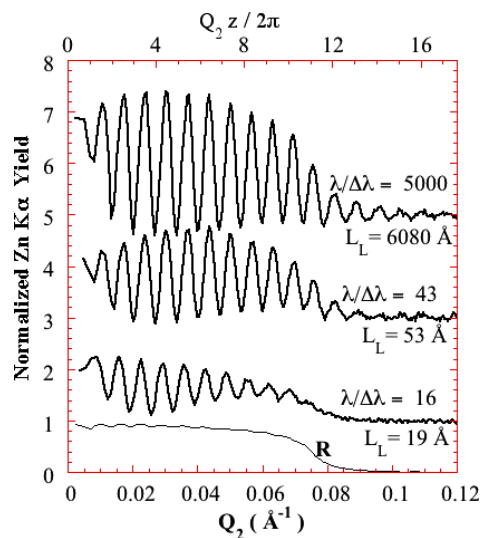


Fig. 5.9. Experimental demonstration of TR-XSW sensitivity to longitudinal coherence (L_L). The bottom and top curves are the Q_2 dependence of the reflectivity (R) and Zn XRF yield for the data from Fig. 6 that was taken with a $Si(111)$ monochromator. The mean Zn height is at $z = 917 \text{ \AA}$. The lower two XRF yield curves are from the same sample, but taken with Si/Mo and C/Rh multilayer monochromators with reduced monochromaticity ($\lambda/\Delta\lambda$) and therefore reduced longitudinal coherence. The top two curves are vertically offset by 2 and 4 units, respectively.

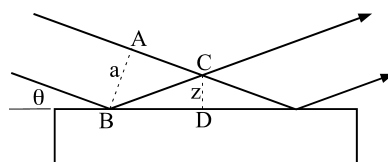


Fig. 5.10. Schematic ray diagram used for illustrating coherence effects between the incident and specular reflected x-ray beams at height z above the mirror surface.

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1 **References**

- 2 1. B. W. Batterman, *Phys. Rev. Lett.* **22** (1969) 703.
- 3 2. P. L. Cowan, J. A. Golovchenko and M. F. Robbins, *Phys. Rev. Lett.* **44**
- 4 (1980) 1680.
- 5 3. J. A. Golovchenko, J. R. Patel, D. R. Kaplan, P. L. Cowan and M. J. Bedzyk,
- 6 *Phys. Rev. Lett.* **49** (1982) 560.
- 7 4. M. J. Bedzyk, G. M. Bommarito and J. S. Schildkraut, *Phys. Rev. Lett.* **62**
- 8 (1989) 1376.
- 9 5. J. Wang, M. J. Bedzyk, T. L. Penner and M. Caffrey, *Nature* **354** (1991) 377.
- 10 6. M. J. Bedzyk, D. H. Bilderback, G. M. Bommarito, M. Caffrey and J. S.
- 11 Schildkraut, *Science* **241** (1988) 1788.
- 12 7. S. I. Zheludeva, M. V. Kovalchuk, N. N. Novikova, A. N. Sosphenov, V. E.
- 13 Erochin and L. A. Feigin, *J. Phys. D* **26** (1993) A202.
- 14 8. W. B. Lin, T. L. Lee, P. F. Lyman, J. J. Lee, M. J. Bedzyk and T. J. Marks,
- 15 *J. Am. Chem. Soc.* **119** (1997) 2205.
- 16 9. J. A. Libera, R. W. Gurney, C. Schwartz, H. Jin, T. L. Lee, S. T. Nguyen,
- 17 J. T. Hupp and M. J. Bedzyk, *J. Phys. Chem. B* **109** (2005) 1441.
- 18 10. M. J. Bedzyk, M. G. Bommarito, M. Caffrey and T. L. Penner, *Science* **248**
- 19 (1990) 52.
- 20 11. J. Wang, M. Caffrey, M. J. Bedzyk and T. L. Penner, *Langmuir* **17** (2001)
- 21 3671.
- 22 12. A. S. Templeton, T. P. Trainor, S. J. Traina, A. M. Spormann and G. E.
- 23 Brown, *Proc. Nat. Acad. Sci.* **98** (2001) 11897.
- 24 13. R. S. Guico, S. Narayanan, J. Wang and K. R. Shull, *Macromolecules* **37**
- 25 (2004) 8357.
- 26 14. B. H. Lin, T. L. Morkved, M. Meron, Z. Q. Huang, P. J. Viccaro, H. M.
- 27 Jaeger, S. M. Williams and M. L. Schlossman, *J. Appl. Phys.* **85** (1999) 3180.
- 28 15. M. Born and E. Wolf, *Principles of Optics*, 6th edn. (Pergamon Press, Oxford,
- 29 1980).
- 30 16. A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment*
- 31 (van Nostrand, New York, 1935).
- 32 17. R. S. Becker, J. A. Golovchenko and J. R. Patel, *Phys. Rev. Lett.* **50**
- 33 (1983) 153.
- 34 18. L. G. Parratt, *Phys. Rev.* **95** (1954) 359.
- 35 19. J. Wang, M. J. Bedzyk and M. Caffrey, *Science* **258** (1992) 775.